Measurements of Thermal Diffusivity for Thin Slabs by a Converging Thermal Wave Technique[1]

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ABSTRACT

The measurement of thermal diffusivity for thin slabs by a converging thermal wave technique has been studied. Temperature variation at the center of the heat source ring that is produced by a pulsed high power laser is detected by an infrared detector. A computer program based on the finite difference method is developed to analyze the thermal diffusivity of the slabs. The materials from the high thermal diffusivity (CVD diamond wafer) to the low thermal diffusivity (stainless steel foil) are used for the measurements. The measurements have been performed by varying the size and the thickness of specimen. The converging thermal wave technique is proved to be a good method to measure the thermal diffusivity of a CVD diamond without breaking the wafer into small specimens. The technique can be applied for a small slab if the diameter of slab is two times larger than that of heat source ring. The sensitivity of thickness in measuring the thermal diffusivity is low for the ordinary CVD diamond. The use of converging thermal wave technique to non-homogeneous, non-uniform and anisotropic materials have been suggested by applying the finite difference method.

KEY WORDS: converging thermal wave technique; CVD diamond; finite difference method; thermal diffusivity

1. INTRODUCTION

As the industry is getting developed, the uses of thin slabs for the thermal management application are growing. For example, thin large area chemical vapor deposition (CVD) diamond wafers are used as a heat spreader and a heat management electrical insulator for high power electronic packages. Diamond has the highest thermal conductivity of known materials above the room temperature. The thermal diffusivity is one of the most important physical properties for the thermal applications. Many techniques have been suggested to measure the thermal diffusivities of thin slabs, such as the 3 omega technique [1], the photo thermal technique [2], the laser flash technique [3], and the dc-heated bar technique [4]. However, these techniques are time consuming in preparation the specimen and in measurements. Some techniques need to cut the wafer to a smaller specimen for the measurement. The measured results of the others vary sensitively with a small change in the thickness of specimen.

A converging thermal wave technique was suggested to overcome the weak points of the others [5, 6]. The main advantage of the converging thermal wave technique is that the thermal diffusivity could be measured without breaking a thin slab into small specimens. Moreover, this technique is a non-contact measuring method, which uses a laser beam as a heat source and uses an IR-detector as a temperature measurement tool. When analyzing the thermal diffusivity from the measured temperature variation, however, the previous studies assumed the specimen as a very thin and infinitely large plate. This assumption may cause an uncertainty for the analysis of relatively thick slabs such as a CVD diamond wafer.

In this study, the temperature variations of a CVD diamond wafer and various metal foils are measured by a converging thermal wave technique. A computer program and an analytic method have been developed to analyze thermal diffusivity from the measurement data without assuming the plate is very thin and infinitely large. The effects of thickness and size of specimen on the applying the converging thermal wave technique have been studied. The possibility of applying this technique not only to the homogeneous, uniform and isotropic material such as a diamond wafer but also to non-homogeneous, non-uniform and anisotropic materials such as a coated material has been suggested.

2. EXPERANTAL APPARATUS AND PROCEDURE

A converging thermal wave technique is adopted to measure the in-plane thermal diffusivity of thin slabs without breaking. A pulsed Nd:Yag laser beam of 0.9 J/pulse, the wavelength of 1.06 μm, the diameter of 10 mm, and the pulse width of 0.35 ms is passed through a convex lens and an axicon to produce an annular ring of heat source. The diameter of the annular heating ring is 9.0 - 10.0 mm on the slab surface and the width of the annular ring is a few tens of micrometers. Temperature excursion at the center of the heating ring is measured with an IR-detector at the rear side of the thin slabs. Dried graphite fluid was sprayed on both surfaces to obtain a uniform emissivity. The measurements are performed on a CVD diamond wafer of thickness of 1mm and on various metal foils of thickness of 50 μm. Detailed experimental procedures are shown in Chae *et al.* [7].

3. MODELING AND ANALYSIS METHOD

3.1 Modeling

Diamond wafer is installed vertically in the air at room temperature. Considering thermal conductivity of air is a few hundred thousandth of that of diamond and the measurements have been performed within a very short time period of 10 ms, the heat loss by air conduction and convection can be neglected. Even the energy density of the heat source ring is large at the moment of laser beam’s irradiation, the density decreases rapidly as the heat diffuses in the wafer. Because the temperature increase of the wafer due to the laser pulse is small, the heat loss from each side of the wafer by the thermal radiation can be neglected. The temperature measurements are affected by the area near the heat source ring, which diameter is 9.0-10.0 mm. Considering the diameter of wafer is 100 mm and the thickness is 1 mm, the wafer can be considered as thin and infinitely large object. Because the properties of the wafer are relatively uniform through the entire wafer, the properties around the heat source ring can be assumed to be constant. Therefore, the heat is assumed to flow in the radial and vertical directions and to not flow in the azimuthal direction.

3.2 Analytic Method

Under the assumptions of section 3.1, the heat conduction equation can be simplified as the following two-dimensional, unsteady-state equation.

$$\frac{1}{a}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \tag{1}$$

where a is the thermal diffusivity, T is the temperature of the wafer, t is time, r is the radius from the center of the heat source ring, and z is the height from the surface where the temperature is measured by an infrared detector (the opposite side of the surface where the laser beam is irradiated). In order to nondimensionalize this equation, the following nondimensional parameters have be chosen.

$$r^* = \frac{r}{R}, \qquad z^* = \frac{z}{e} \qquad t^* = \frac{at}{R^2} \tag{2}$$

where R is the radius of the heat source ring and e is the thickness of the wafer. Let $A = e/R$, then a nondimensionalized governing equation is obtained as

$$\frac{\partial T}{\partial t^*} = \frac{\partial^2 T}{\partial r^{*2}} + \frac{1}{r^*}\frac{\partial T}{\partial r^*} + \frac{1}{A^2}\frac{\partial^2 T}{\partial z^{*2}} \tag{3}$$

The boundary conditions and the initial conditions are

$$\frac{dT}{dz^*} = 0 \qquad \text{at } z^* = 0 \text{ and } z^* = 1 \qquad \text{for } t^* > 0 \tag{4}$$

$$T \text{ is finite} \qquad \text{at } r^* = 0 \qquad \text{for } t^* > 0 \tag{5}$$

$$\begin{aligned} T(r^*, z^*, 0) &= T_0 \qquad \text{at } r^* = 1 \text{ and } z^* = 1 \\ &= 0 \qquad \text{at } r^* \neq 1 \text{ and } z^* \neq 1 \end{aligned} \tag{6}$$

where T_0 is the initial temperature at the heat source ring.

Solving the governing equation (3) with the boundary and initial conditions of (4)-(6) by applying the separation of variables, the temperature distribution of the wafer with time is obtained as following equations [8].

$$T(r^*, z^*, t^*) = T_0 \times \frac{1}{2t^*} \exp(-\frac{1+r^*}{4t^*}) \times I_0(\frac{r^*}{2t^*}) \times \left\{ 1 + \sum_{n=1}^{\infty} 2\cos h_n \cos h_n z^* \exp(-\frac{h_n^2}{A^2} t^*) \right\} \tag{7}$$

The temperature at the origin ($r^* = 0$, $z^* = 0$), where the temperature is measured by an infrared detector, can be simplified as

$$T(0,0,t^*) = T_0 \times \frac{1}{2t^*} \exp(-\frac{1}{4t^*}) \times \left\{ 1 + \sum_{n=1}^{\infty} 2\cos np \exp(-\frac{n^2 p^2}{A^2} t^*) \right\} \tag{8}$$

3.3 Finite Difference Method

The governing equation (1) is a two-dimensional parabolic equation. Many numerical methods have been suggested to solve the two-dimensional parabolic equation problems, but among them the alternating direction implicit (ADI) method is the most frequently used because it is non-conditionally stable [9]. ADI method is applied on the equation (1). A time step is divided by two, and for the first half time step, the following finite difference equation is applied in the horizontal direction.

$$\frac{T_{i,j}^{n+\frac{1}{2}} - T_{i,j}^{n}}{a\Delta t/2} = \frac{1}{r_i} \frac{T_{i+1,j}^{n+\frac{1}{2}} - T_{i-1,j}^{n+\frac{1}{2}}}{2\Delta r} + \frac{T_{i-1,j}^{n+\frac{1}{2}} - 2T_{i,j}^{n+\frac{1}{2}} + T_{i+1,j}^{n+\frac{1}{2}}}{\Delta r^2} + \frac{T_{i,j-1}^{n} - 2T_{i,j}^{n} + T_{i,j+1}^{n}}{\Delta z^2} \tag{9}$$

Then, for the second half time step, the following equation is applied in the vertical direction.

$$\frac{T_{i,j}^{n+1} - T_{i,j}^{n+\frac{1}{2}}}{a\Delta t/2} = \frac{1}{r_i} \frac{T_{i+1,j}^{n+\frac{1}{2}} - T_{i-1,j}^{n+\frac{1}{2}}}{2\Delta r} + \frac{T_{i-1,j}^{n+\frac{1}{2}} - 2T_{i,j}^{n+\frac{1}{2}} + T_{i+1,j}^{n+\frac{1}{2}}}{\Delta r^2} + \frac{T_{i,j-1}^{n+1} - 2T_{i,j}^{n+1} + T_{i,j+1}^{n+1}}{\Delta z^2} \tag{10}$$

At the center of the heat source ring (r=0), the first terms of the right hand side of equations (9) and (10) are $\frac{1}{0}$ and become singular. Therefore, equations (9) and (10) cannot be applied at the meshes of r=0. In order to detour the singularity problem, the following is applied to the equation (1).

$$\left(\frac{1}{r}\frac{\partial T}{\partial r}\right)_{r=0} = \frac{\frac{\partial}{\partial r}\frac{\partial T}{\partial r}}{\frac{\partial}{\partial r}(r)} = \left(\frac{\partial^2 T}{\partial r^2}\right)_{r=0} \tag{11}$$

Then the equation (1) becomes

$$\frac{1}{a}\frac{\partial T}{\partial t} = 2\frac{\partial^2 T}{\partial r^2} + \frac{\partial^2 T}{\partial z^2} \tag{12}$$

Equation (12) is discretized with the same method as equations (9) and (10), and is applied at the origin.

The area where temperature varies due to the heat irradiated from the laser beam is expanded continuously as time goes. However, the computational area cannot be infinitely large. The computational area is determined up to the area where heat is spread during the consideration time period. The end of computational area is assumed to be insulated. In the real calculation, the radius of computational area is about three times larger than that of the heat source ring.

4. RESULTS AND DISCUSSION

4.1 CVD Diamond Wafer

The heat expansion in the CVD diamond wafer just after laser beam is irradiated on the heat source ring is calculated by the finite difference method, and the results are shown in Fig. 1. Before the laser beam is irradiated, the temperature of whole wafer is assumed to be 0. At t=0, the laser is irradiated and the temperature of heat source ring (r=4.4775 mm, z=1 mm) is assumed to become 1. The thermal diffusivity of the wafer is assumed to be 690 mm^2s^{-1}. The absolute value of temperature varies with the size of mesh, but the characteristics of the temperature distribution do not varies even the size of mesh varies. The temperature distribution in the circular area of 17.91 mm in radius was calculated. The figure shows temperature distributions of every 0.2 ms from 0.2 ms to 1.0 ms. When 0.2 ms passed after laser beam is irradiated, the isothermal lines are formed in concentric circular shape, center of which is heat source ring. It can be known that the heat is spread uniformly in the radial and the thickness directions. As the time passes, the heat spread to the inner direction and the outer direction. The diffusion velocity in the inner direction is faster than that in the outer direction. It is because the cross section decreases as the heat spread in the inner direction, but it increases as the heat spread in the outer direction. At 0.8 ms, the isothermal lines are shown as almost vertical lines. After this time, most of heat is spread in the radial direction, and temperature is almost uniform in the thickness direction.

The temperature distributions of the top surface of the wafer are calculated by the finite difference method, and the results are shown in Fig. 2. The curve is drown in every 1 ms from 1 ms to 10 ms. The *x* coordinate is the distance from the center of heat source ring, and *y* coordinate represents temperature. Because the temperature is known to be uniform in the vertical direction after 0.8 ms from the previous figure, the temperature distributions in the Fig. 2 represent that of the same radius. The curve that shows the highest value at the heat source ring is the temperature distribution at the time of 1 ms. The peak value of the temperature is reduced rapidly from the value of 1 at t=0 to the value of 0.004 at t=1 ms. The point of the highest temperature moves one mesh inner side, from 4.4775 mm to 4.253625 mm. This represent that the heat diffusion rate to inner direction is faster than that to outer direction. Until this time, heat does not reach to the center, and the temperature of the center does not vary. As the time passes, the point where the highest temperature occurs moves to the inner direction, and the peak value reduces gradually and the curve becomes blunt curves. The heat diffused to the outside expands continuously. However, the heat diffused to the inner direction accumulated at the center, and the temperature begins to increase rapidly. The thick curve in the figure represents the temperature distribution at the time of 7 ms. The temperature at the center reaches at the highest value at this time, and the temperature at the center is the highest among the whole wafer. (The exact time at this phenomenon is

7.28 ms.) As the time passes the heat diffused to the inner side of the heat source ring spreads to the outer direction, and the temperature at the center decreases gradually.

The time variations of temperature at the center are calculated by the finite difference method and by the analytic method. Figure 3 shows the temperature distributions with the measurement data. The *x* coordinate represents the time after the laser beam is irradiated, and the *y* coordinate represents the temperature of the center, which is normalized by the peak value at center. The thick solid curve is the temperature distribution that is calculated by the finite difference method for the case of wafer thickness of 1mm, radius of heat source ring of 4.4775 mm, and the thermal diffusivity of 690 mm^2s^{-1}. Calculation has been performed for the period until the heat reached to the outer meshes. There was almost no temperature variation at the center of the heat source ring until the heat irradiated from the laser reaches to the center. However, the temperature increases very rapidly after heat reaches to the origin. The temperature reaches at its maximum value at 7.28 ms, and then the temperature at the center decreases slowly as the heat spreads to the surroundings. The dashed line is the temperature variation calculated by the analytic methods of equation (8) for the same parameters with the finite difference method. The temperature variations obtained by these two methods agree quite well. The times for the peak temperature at the center are 7.28 ms and 7.16 ms, respectively, and shows only 1.6 % difference. The measurement data shown in the figure is the temperature variation at the center of the heat source ring of radius 4.4775 mm detected by the IR detector for the wafer of thickness of 1 mm. Because the measurement data agrees well with the predicted data by finite difference method or by the analytic method, the heat diffusivity of the wafer can be known to be 690 mm^2s^{-1}.

Comparing temperature time variation measurement results to that predicted by the finite difference method by trial and error method, the best matched thermal diffusivity is determined for the thermal diffusivity of the diamond wafer. The thermal diffusivities of a whole CVD diamond wafer are measured by this converging thermal wave technique. The detailed results are shown in Chae *et al.* [7].

The sensitivity of thickness for the analysis of thermal diffusivity has been studied. For a typical value of thermal diffusivity of a CVD diamond wafer, 600 mm^2s^{-1}, the time for peak value has been calculated by the finite difference method with varying the thickness of wafer. The time for peak value is appeared increasing if the wafer thickness is larger than 2.5 mm. Thus, if the thickness is smaller than 2.5 mm and the thermal diffusivity is larger than 600 mm^2s^{-1}, the thickness change cause almost no effect on the analysis of the thermal diffusivity. Because the thickness of common CVD diamond wafers are about 1 mm, the converging thermal wave technique gives reliable results even if the thickness of wafer varies a little.

4.2 Metal Foils

The thermal diffusivities of copper, aluminum, nickel, and stainless steel foils are measured by the converging thermal wave technique. Figure 4 compared the measured thermal diffusivities with the diffusivities obtained in the literatures. The thicknesses of foils are 50 μm and the purities are above 99.99 %. The metal foils of high thermal diffusivity such as copper, aluminum and nickel show good agreement with the literature values with the difference of 3~8 %. However, stainless steel, the low thermal diffusivity metal, shows a relatively large difference of 15 %. One of the reasons is believed to be that because the measurement time of the low thermal diffusivity metal takes longer than that of the high thermal diffusivity metals, the assumption of no heat loss from the sides of foil may cause larger error. Because the measurement of CVD diamond wafer are finished within much less time period than the metal foils are, the error due to the heat loss from the sides of the diamond wafer will be much smaller. The time for peak value of a stainless steel is about 1450 ms, but that of CVD diamond is 7 ms.

The thermal diffusivities of various specimen sizes are measured and calculated by the finite difference method. Figure 5 shows the time for peak value of copper foils, which radius are reduced from 20 mm to 8.5 mm. The radius of heat source ring is 5.0

mm. Until the radius of specimen is reduced up to two times larger than that of heat source ring, the peak time does not varies significantly. However, when the radius of specimen is reduced to 1.7 times of that of heat source ring, the peak time increases. The peak times predicted by the finite difference method show good agreement with the measured values. The peak time increase very rapidly when the radius is reduced to smaller than 1.9 time of heat source ring. The converging thermal wave technique is known to be able to apply for a thin and large area slabs. However, from this result it is known that if the diameter of specimen is at least two times larger than that of heat source ring, the converging thermal wave technique gives a reasonable value of thermal diffusivity.

The time for peak value increases rapidly when the heat source ring approaches the edge of specimen. The increase of time for peak value means the decrease of thermal diffusivity. This result can be used to detect a possible crack in a CVD diamond wafer, which is hard to find in naked eyes. If a region in the diamond wafer shows sudden decrease of thermal diffusivity, it means a crack may exist near the heat source ring. By drawing the thermal diffusivity contour map, the position of crack can be detected.

The time variations of temperature at off-center points of the heat source ring are measured and calculated by the finite difference method. Figure 6 shows the temperature variations of some of the points, the center (r=0 mm), a point inside of heat source ring (r=3 mm) and a point outside of heat source ring (r=9 mm). The *x* coordinate represents time and the *y* coordinate represents temperature normalized by the peak temperature at center. The figure shows that the measured and calculated values agree well.

4.3 Comparison of the Analyses by the Analytic Method and by the Finite Difference Method

The analytic method has the advantage of simple calculation, but it can be applied for the plates of infinitely large area, only. The temperature at the center can be obtained relatively easily, but the temperature distribution at the other points is very difficult to obtain.

On the other hand, the finite difference method can be applied for any size specimens. It can calculate temperature of the specimen at any point relatively easily. Moreover, by changing the material properties of each node, the finite difference method can be applied for the analysis of thermal diffusivity of non-homogeneous, non-uniform and anisotropic materials such as a composite material. It also can be applied for a specimen of two layered materials, such as a metal plate coated with a diamond-like carbon.

5. CONCLUSION

A converging thermal wave technique has been adopted to measure the thermal diffusivity of thin slabs. The technique is proved to be a good method to measure the thermal diffusivity of a CVD diamond without breaking the wafer into small specimens.

A finite difference method model was developed for the analysis of the measurement results. By applying the finite difference method, it is proved that the converging thermal wave technique can be applied for a thin small slab as long as the diameter of slab is two times larger than that of heat source ring. The sensitivity of thickness in measuring the thermal diffusivity is proved to be very small for the ordinary CVD diamond wafers. By applying the finite difference method, the use of converging thermal wave technique not only to homogeneous uniform isotropic materials but also to non-homogeneous non-uniform anisotropic materials are suggested.

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FIGURE CAPTIONS

Fig. 1. Temperature profile of the wafer calculated by the finite difference method. The laser beam is irradiated at heat source ring (r=4.4775mm, z=1mm) at t=0.

Fig. 2. Temperature distribution of the top of the wafer calculated by the finite difference method.

Fig. 3. Temperature variations at the center of the wafer obtained by the finite difference method, the analytic method, and the measurements.

Fig. 4. Thermal diffusivities of various metal foils measured by the converging thermal wave technique and obtained in literatures. The foil thickness is 50μm.

Fig. 5. Time for peak value of various size copper foils measured by the converging thermal wave technique and that calculated by the finite difference method.

Fig. 6. Temperature variation of different points of a copper foil.

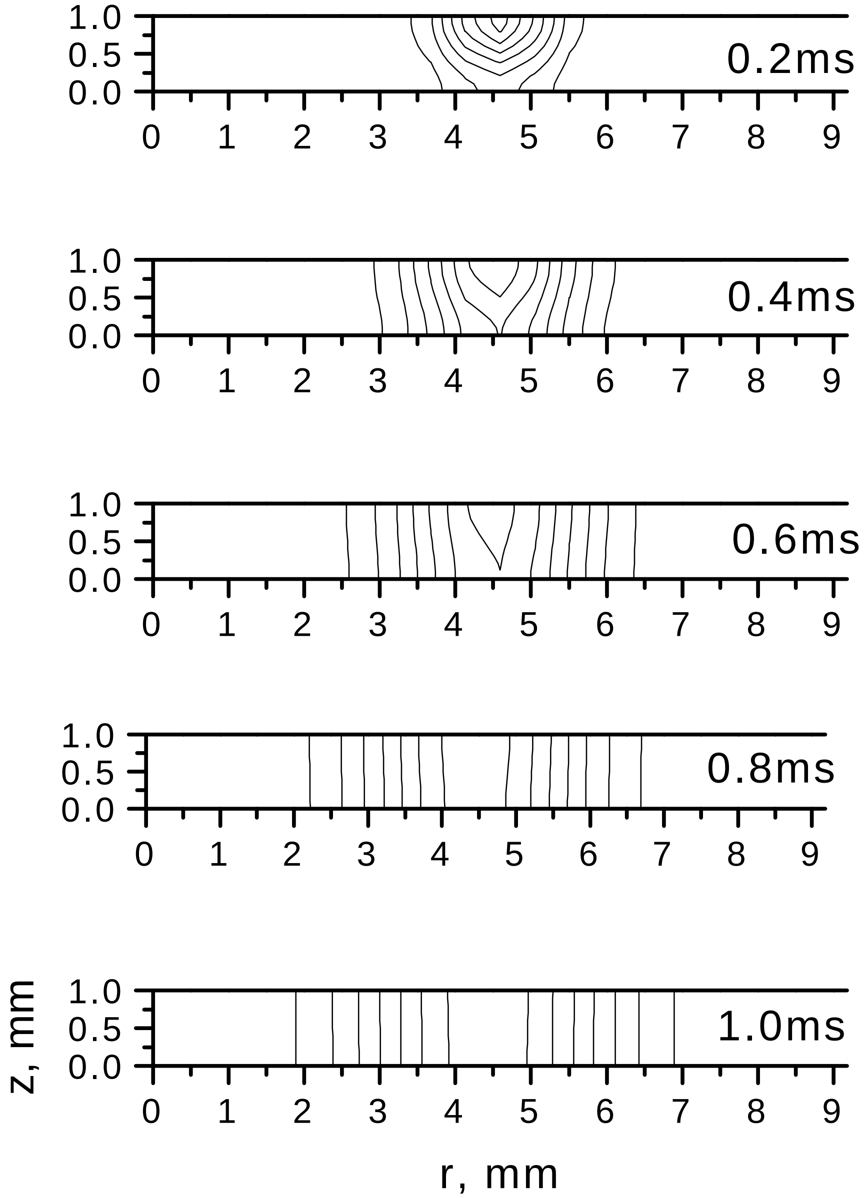


Fig. 1

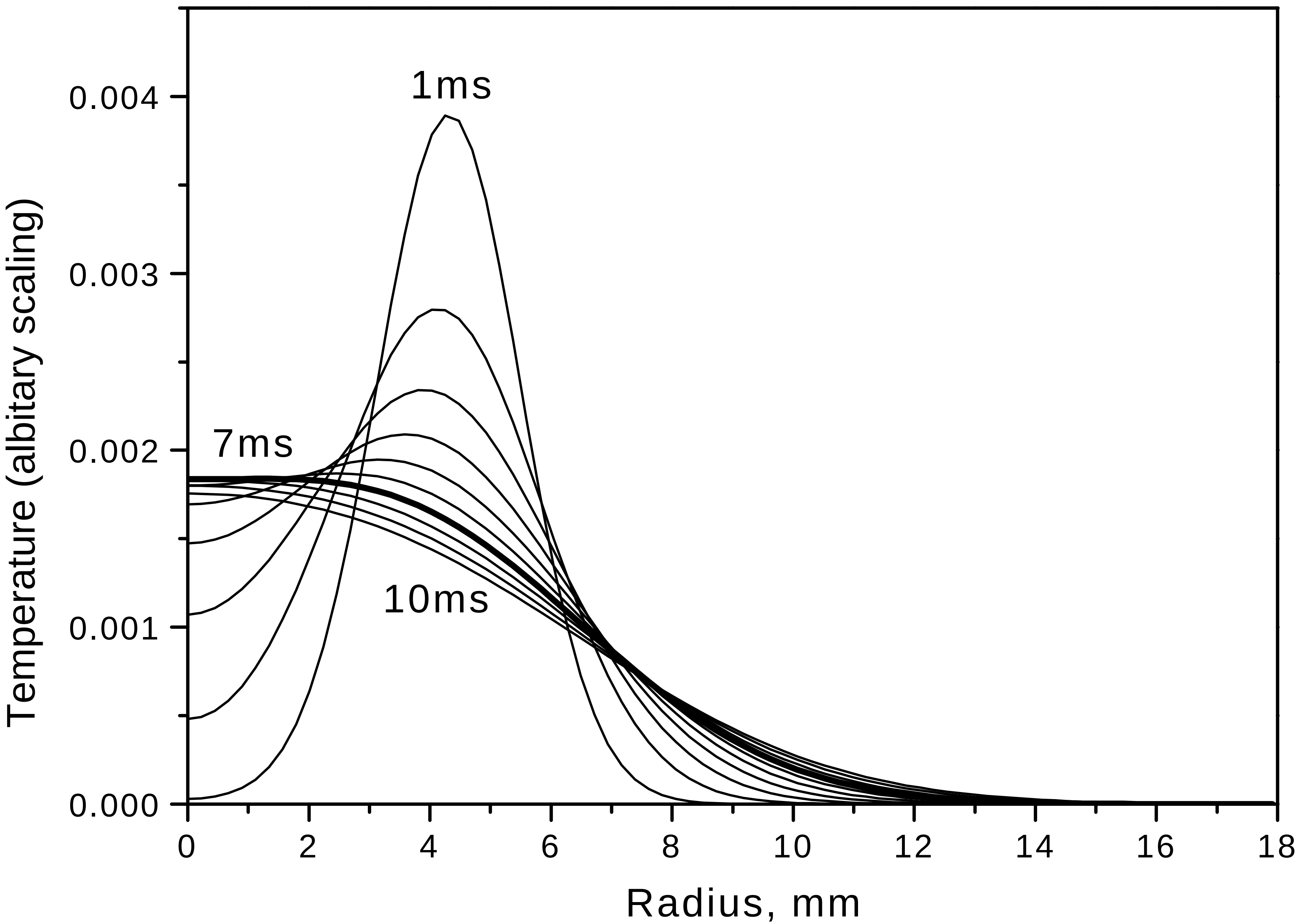

1ms
7ms
10ms
0.000
0.001
0.002
0.003
0.004
Temperature (albitary scaling)
0
2
4
6
8
10
12
14
16
18
Radius, mm

Fig. 2

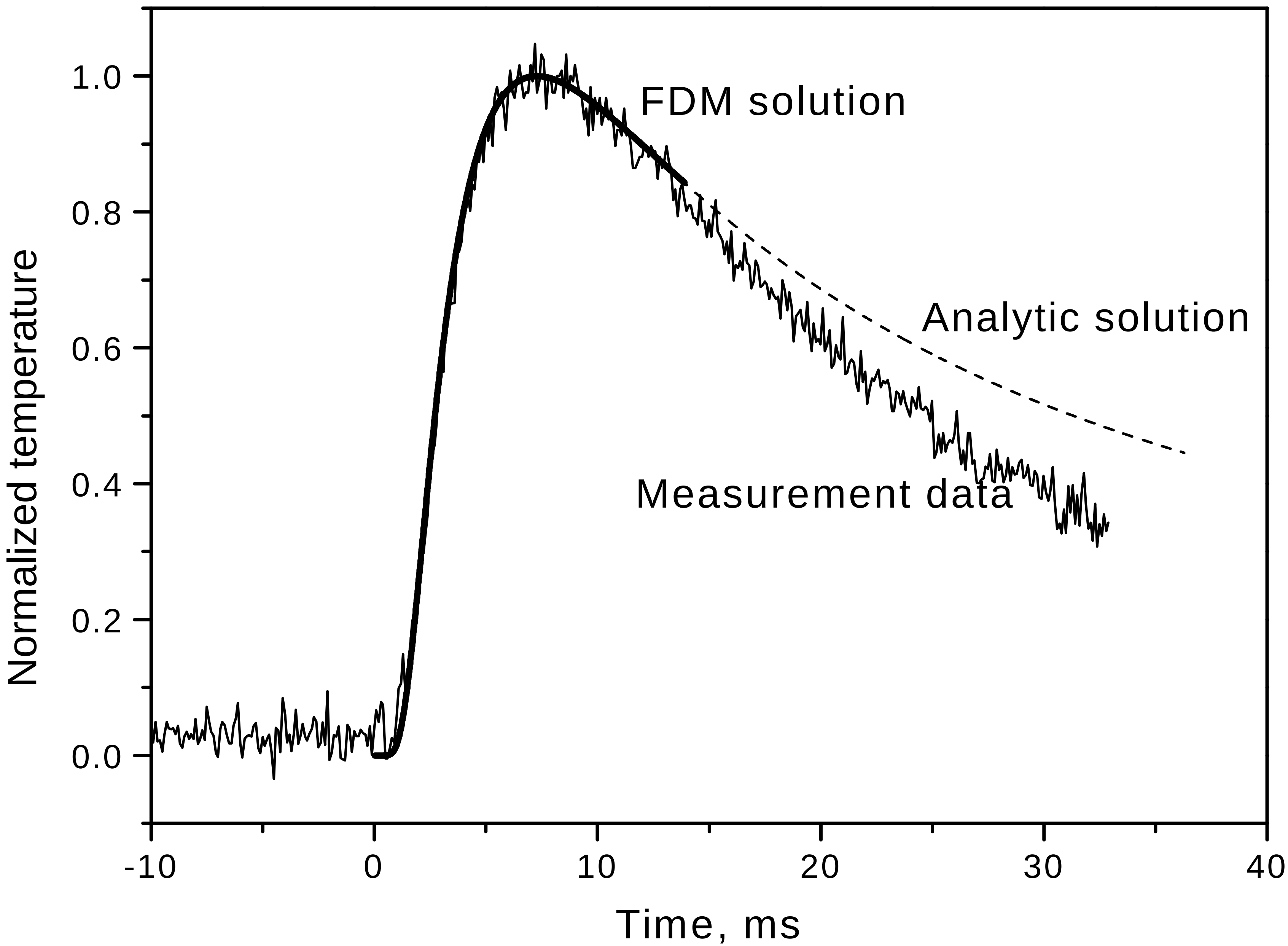

FDM solution
Analytic solution
Measurement data
Normalized temperature
Time, ms
1.0
0.8
0.6
0.4
0.2
0.0
-10
0
10
20
30
40

Fig. 3

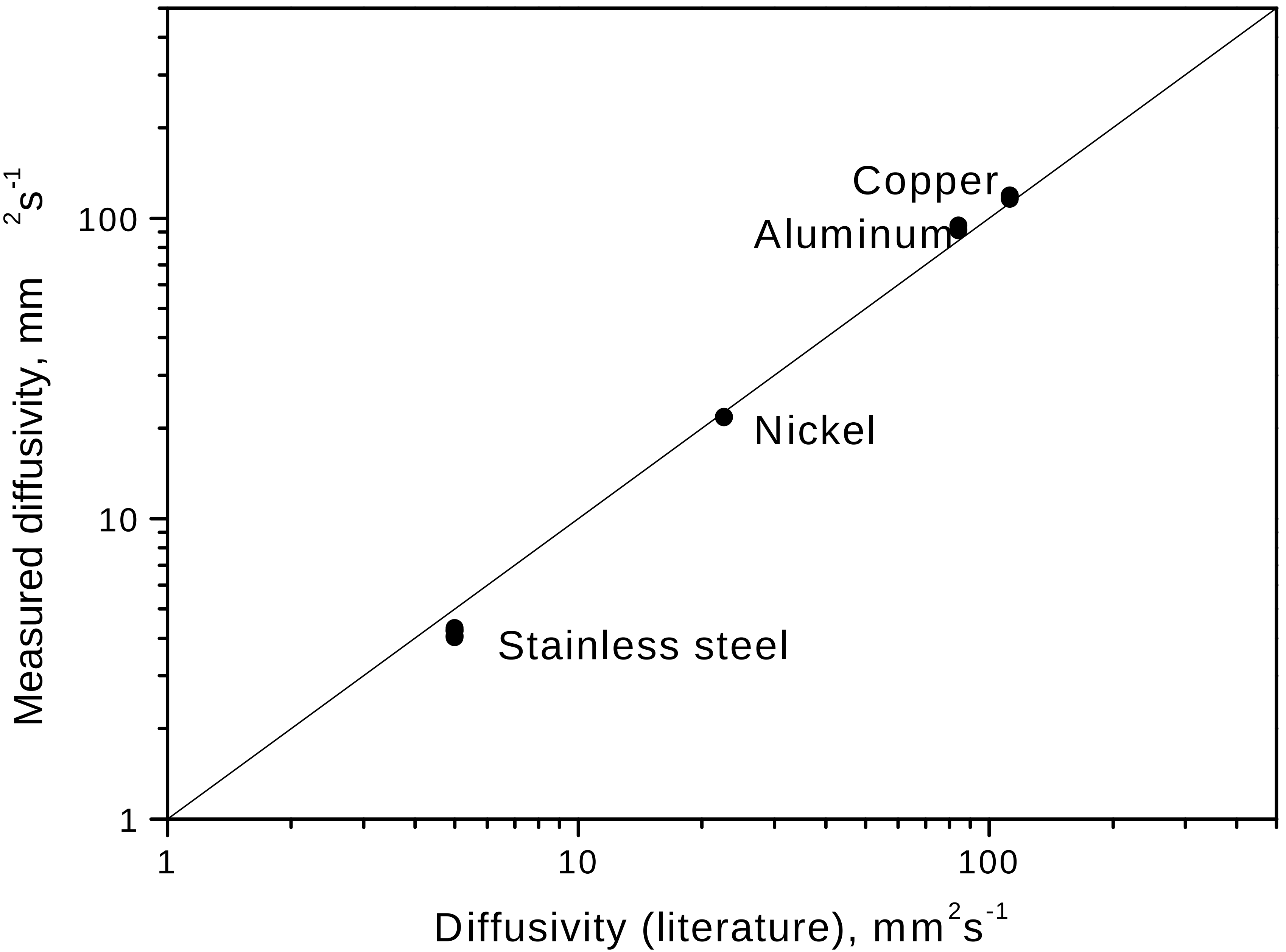
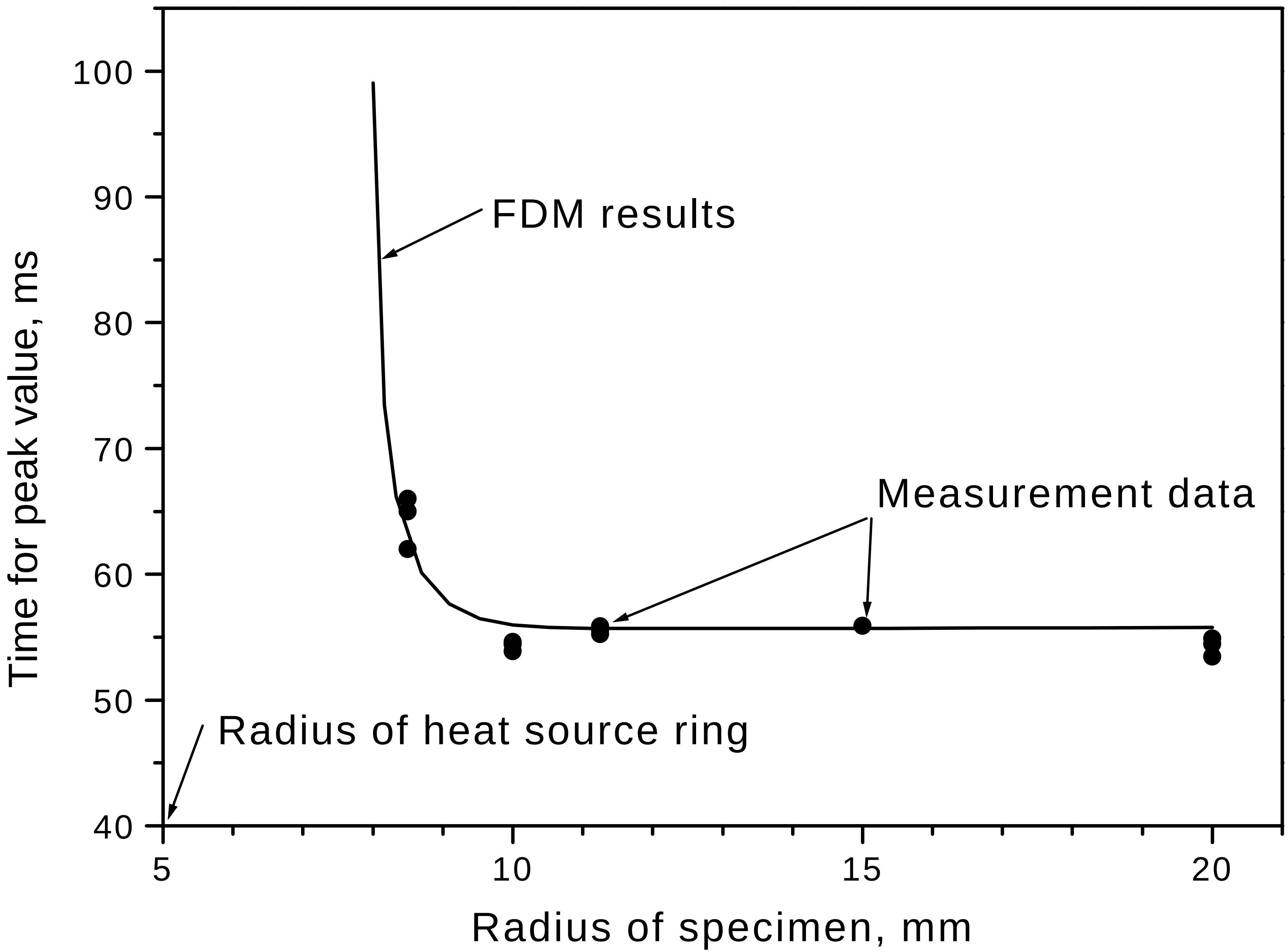
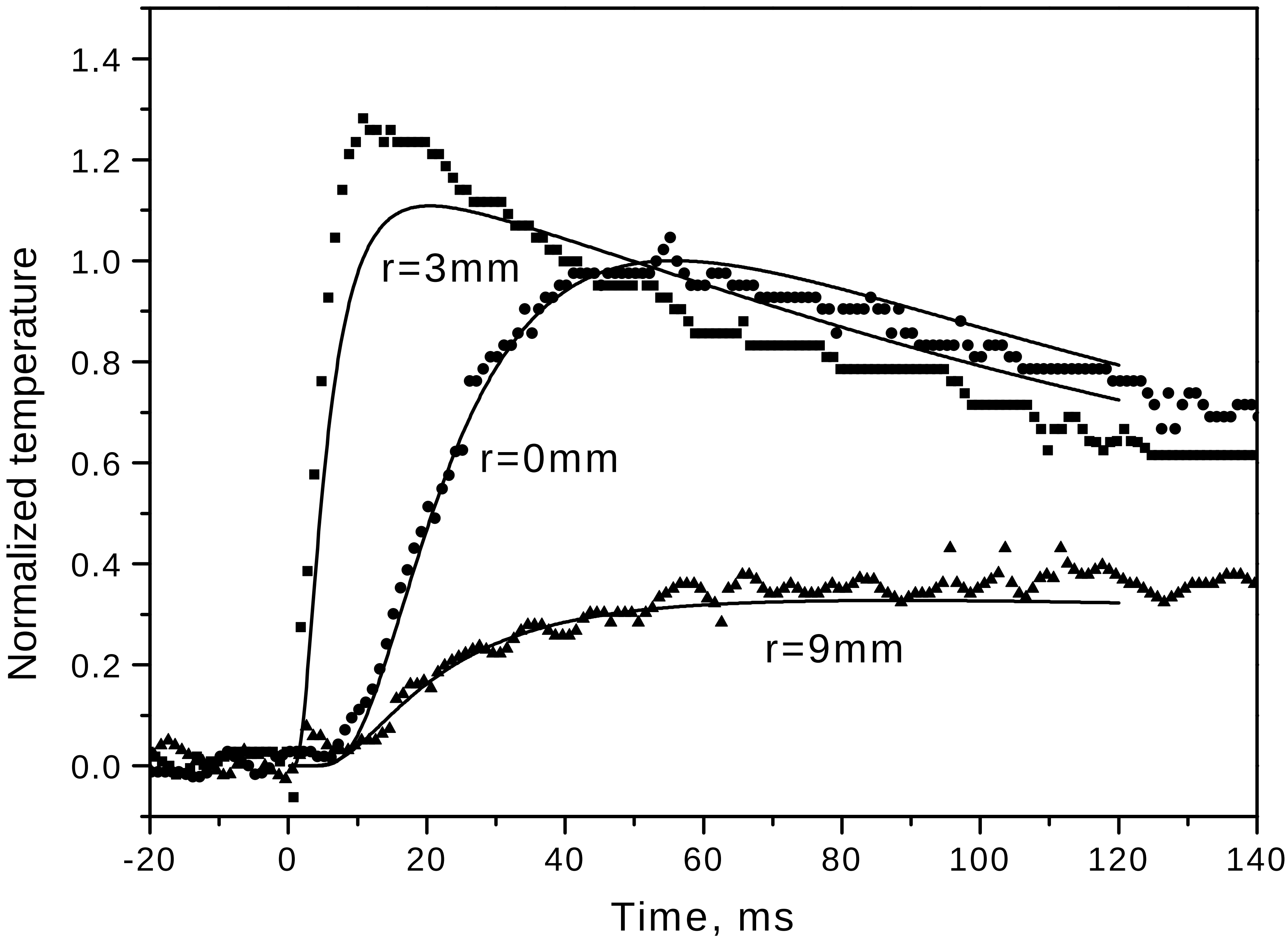

Copper
Aluminum
Nickel
Stainless steel
100
10
1
1
10
100
Measured diffusivity, mm^{2}s^{-1}
Diffusivity (literature), mm^{2}s^{-1}

Fig. 4



Time for peak value, ms
100
90
80
70
60
50
40
5
10
15
20
Radius of specimen, mm
FDM results
Measurement data
Radius of heat source ring

Fig. 5



1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
Normalized temperature
-20
0
20
40
60
80
100
120
140
Time, ms
r=3mm
r=0mm
r=9mm

Fig. 6